Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT2 **Branch: B. Tech (All)**

Semester : 1 **Date : 28/11/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $y=\frac{1}{x}$ then y_n equal to

(A) $\frac{(-1)^n n!}{x^n}$ (B) $\frac{(-1)^n n!}{x^{n+1}}$ (C) $\frac{(-1)^{n-1}(n-1)!}{x^n}$ (D) None of these

b) If $y=(10)^{2x}$ then y_n equal to

(A) $2^n(10)^{2x}(\log 10)^n$ (B) $2^n(10)^{2x}(\log 2)^n$ (C) $2^n(10)^{2x}(\log 10)^{n+1}$

(D) none of these

c) The series $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+.....$ represent expansion of

(A) $\cot^{-1}x$ (B) $\tan^{-1}x$ (C) $\sin^{-1}x$ (D) $\sin x$

d) If $y=\log(1+x)$, then x equal to

(A) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}$ (B) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$ (C) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+\frac{y^4}{4!}+.....$

(D) none of these

e) $\lim_{x\to\infty}\frac{x^n}{e^x}=$ ______

(A) 0 (B) 1 (C) 2 (D) none of these

f) $\lim_{x\to 0}\frac{5^x-3^x}{x}=$ ______

(A) 2 (B) $\log 2$ (C) $\log 15$ (D) $\log\left(\frac{5}{3}\right)$

g) If $P=r\tan\theta$, then $\frac{\partial P}{\partial r}$ is equal to

(A) $\sec^2\theta$ (B) $\tan\theta$ (C) $\tan\theta+r\sec^2\theta$ (D) $\frac{1}{2}\tan\theta$

h) If $x = r\cos\theta,\ y = r\sin\theta,$ then $\dfrac{\partial r}{\partial x}$ is equal to

(A) $\sec\theta$ (B) $\sin\theta$ (C) $\cos\theta$ (D) $\cos ec\theta$

i) If $f(x,y)=0$, then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{\partial f/\partial x}{\partial f/\partial y}$ (B) $\dfrac{\partial f/\partial y}{\partial f/\partial x}$ (C) $-\dfrac{\partial f/\partial y}{\partial f/\partial x}$ (D) $-\dfrac{\partial f/\partial x}{\partial f/\partial y}$

j) If $\dfrac{\partial(u,v)}{\partial(x,y)} \times \dfrac{\partial(x,y)}{\partial(u,v)}$ is equal to

(A) 1 (B) – 1 (C) zero (D) none of these

k) If $y = \cos\theta + i\sin\theta$, then the value of $y + \dfrac{1}{y}$ is

(A) $2\cos\theta$ (B) $2\sin\theta$ (C) $2\cos ec\theta$ (D) $2\tan\theta$

l) If $x_r = \cos\left(\dfrac{\pi}{2^r}\right) + i\sin\left(\dfrac{\pi}{2^r}\right)$ then $x_1x_2x_3.....$ to ∞ is

(A) – 3 (B) – 2 (C) – 1 (D) 0

m) An eigenvalue of a square matrix A is $\lambda = 0$. Then

(A) $|A| \neq 0$ (B) A is symmetric (C) A is singular (D) A is skew-symmetric

n) The product of the eigenvalues of $\begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$ is

(A) 2 (B) 4 (C) 6 (D) 0

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $y = \dfrac{x}{x^2 + a^2}$ then find y_n. **(5)**

b) Prove that $(1+x)^x = 1 + x^2 - \dfrac{1}{2}x^3 + \dfrac{5}{6}x^4 - .....$ **(5)**

c) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then show that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}\right)^2 u = -\dfrac{9}{(x+y+z)^2}$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) If $y = \sin\left(m\sin^{-1}x\right)$ then prove that **(5)**

$\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0$.

b) Prove that $\cos^{-1}\left[\tanh(\log x)\right] = \pi - 2\left(x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - .....\right)$ **(5)**

c) Evaluate: $\lim\limits_{x\to 0}\left(\dfrac{a}{x} - \cot\dfrac{x}{a}\right)$ **(4)**

Q-4 **Attempt all questions** **(14)**

a) Evaluate: $\lim\limits_{x\to 0}\left(\dfrac{a^x + b^x + c^x + d^x}{4}\right)^{\frac{1}{x}}$ **(5)**

b) If $u=\frac{y^2}{x}$, $v=\frac{x^2}{y}$, evaluate $J=\left(\frac{x,y}{u,v}\right)$ and $J'=\left(\frac{u,v}{x,y}\right)$ and hence verify that $JJ'=1$. **(5)**

c) Expand $f(x)=x^4-11x^3+43x^2-60x+14$ in powers of $(x-3)$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) If $u=\tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$. **(5)**

b) Evaluate: $\lim_{x\to\frac{\pi}{4}}(1-\tan x)\sec 2x$ **(5)**

c) Find nth derivative of $\tan^{-1}x$. **(4)**

Q-6 **Attempt all questions** **(14)**

a) Using the formula $R=\frac{E}{I}$, find the maximum error and percentage of error in R if $I=20$ with a possible error of 0.1 and $E=120$ with a possible error of 0.05 and $R=6$. **(5)**

b) Prove that $(a+ib)^{\frac{m}{n}}+(a-ib)^{\frac{m}{n}}=2\left(a^2+b^2\right)^{\frac{m}{2n}}\cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$. **(5)**

c) Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}-3 & -7 & -5\\ 2 & 4 & 3\\ 1 & 2 & 2\end{bmatrix}$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Reduce the matrix $A=\begin{bmatrix}1 & -1 & 2 & -3\\ 4 & 1 & 0 & 2\\ 0 & 3 & 1 & 4\\ 0 & 1 & 0 & 2\end{bmatrix}$ to the normal form and find its rank. **(5)**

b) Using De Moivre's theorem prove that **(5)**

(i) $\cos 5\theta=5\cos\theta-20\cos^3\theta+16\cos^5\theta$

(ii) $\sin 5\theta=5\sin\theta-20\sin^3\theta+16\sin^5\theta$

c) If $\tan(\alpha+i\beta)=x+iy$ then prove that $x^2+y^2+2x\cot 2\alpha=1$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Investigate for what values of λ and μ the equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(5)**

b) Find the continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. **(5)**

c) Find the inverse of $A=\begin{bmatrix}-1 & 1 & 2\\ 3 & -1 & 1\\ -1 & 3 & 4\end{bmatrix}$ by Gauss-Jordan reduction method. **(4)**